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# Modeling and Measuring the Thermal Response of Aluminum Spheres with Convection Dominating

## **Abstract:**

Temperature is a very common quantity that many different types of engineers aim to measure accurately over large ranges and thermocouples are the go-to device for this job. Convection is also a complicated phenomenon that must be considered when designing cooling solutions to many different systems. This examination focuses on accurately modeling the convective cooling of an aluminum sphere with the lumped thermal capacity model and experimentally determining the value of the convective heat transfer coefficient using thermocouples as the measuring devices. The convective coefficients for the spheres undergoing natural and forced convection were calculated to be  $330 \pm 30 \frac{W}{m^2K}$  and  $1.6 \times 10^3 \pm 400 \frac{W}{m^2K}$  respectively by submerging them in 0 °C ice-water and logging an attached thermocouple's voltage response. Objects appear to cool quicker when the fluid they are submerged in is flowing, possibly due to the faster bulk movement of particles. Thermocouples and the lumped thermal capacity model are extraordinarily useful and accurate tools when used correctly and have many applications when designing cooling solutions in the steel, aerospace, and power industries to name a few.

## **Introduction:**

Thermal considerations are crucial when designing effective cooling solutions to real-world engineering problems ranging from cooling AMD's new 7nm CPUs to making a research outpost habitable in arctic sub-zero conditions. Having accurate mathematical models and temperature measuring devices for the physical phenomenon of cooling by convection makes an iterative design process much faster and more effect. If one is undertaking these or similar endeavors, it is important to understand basic convection models and temperature measuring techniques.

The first law of thermodynamics states that if there are any changes in the total amount of energy, work, or heat of a closed system, then all these quantities must balance so that the system has a net energy change of zero. It is essentially a restatement of the conservation of energy principle so widely used throughout physics. Looking at physical phenomenon through this lens allows energy balances to be written, yielding governing partial-differential equations that can be solved for a temperature response that is a function of position and time.

There are three basic modes of heat transfer: radiation, conduction, and convection. Radiation occurs when objects emit electromagnetic radiation as a function of their temperature raised to the fourth power and can happen even without any participating medium to transport the heat (in a vacuum). Conduction occurs within a solid object and is essentially how heat diffuses through a solid medium. Convection is the transferring of heat due to the movement of a fluid (gas or liquid) around an object<sup>1</sup>.

Newton's Law of cooling can be used to obtain an equation for the heat lost by a body from convection. Combining this with the first law of thermodynamics can be used to model the cooling of a system in which convection is dominating—this is an important assumption that can be checked later by using a quantity known as the Biot number. This system of modeling a cooling problem that neglects radiation and conduction (called the lumped thermal capacity model) will be examined and discussed in more depth later. A constant called the convective heat transfer coefficient,  $h$ , is a complex quantity that is directly proportional to the rate of heat transfer from convection that an object is encountering<sup>2</sup>. This value depends on the solid object's geometry, the fluid's properties, and the solid's properties, therefore the best way to obtain it is usual through experimentation due to its complex dependencies. With a model in hand, it is important to be able to conduct experiments to validate the model. For this, an accurate way of measuring temperatures is crucial.

Thermocouples consist of two strands of different metal materials joined together at both ends, creating two junctions (a hot and cold one). The types of metals used determine the type of thermocouple (K-Type, J-Type, T-Type, E-Type, etc.). Each type has certain advantages such as their useful temperature range and certain disadvantages such as their accuracies (this information is readily found online). Temperature is measured at one junction site by taking advantage of what is known as the Seebeck effect (also known as the thermoelectric effect)—a phenomenon that results in two different electrical conductors creating a voltage difference that is a function of the temperature the thermocouple is measuring<sup>4</sup>. This phenomenon leads to the generation of energy and the thermocouple need not be powered by an external source. One end of the thermocouple must be put in a temperature that is known however (reference temperature), since this will affect the voltages that the thermocouple is generating. A common choice is to reference the thermocouple to 0 °C (ice-water), and look up the voltage vs temperature curves for the type of thermocouple being used referenced to 0 °C. The sensitivity can then be calculated and used to convert measured voltage differences into temperatures.

These techniques are used every day by engineering professionals all over the world to obtain accurate experimental data for cooling phenomenon. The following work aims to explore a concrete method for modeling and measuring heat transfer when convection is dominating. Heated aluminum spheres were submerged in an ice-bath and their thermal responses were obtained by using voltage data gathered from calibrated K-type thermocouples. The convective heat transfer coefficients that are directly proportional to the rate of heat transfer occurring were then calculated and compared for a stagnant (natural/free convection) and flowing (forced convection) surrounding fluid. How does the fluid's velocity effect the rate of heat loss occurring?

# Results:

## Manuel Measurements

A spherical aluminum ball with an embedded k-type thermocouple was heated to 90 °C and then suddenly submerged in a bath of 0 °C stagnant ice-water, allowing natural convection to cool the sphere. The thermocouple was physically referenced to 0 °C by submerging and allowing a reference junction to come to equilibrium in the ice bath. A digital multimeter was used to measure the voltage difference between the two thermocouple junctions. The voltage at 5 second increments was recorded by hand and then converted to temperature by using tables for k-type thermocouples after removing any offsets and calibrating as described in more detail in the methods section. The measured temperature response for three trials is plotted below in figure 1. By modeling the cooling with the lumped thermal capacity model (equations 4,5, and 7), the biot number was found to be  $.046 \pm .003$  by using equation 8 to fit the experimental data. The natural convective heat transfer coefficient was then calculated to be  $370 \pm 30 \frac{W}{m^2K}$  using equation 6.

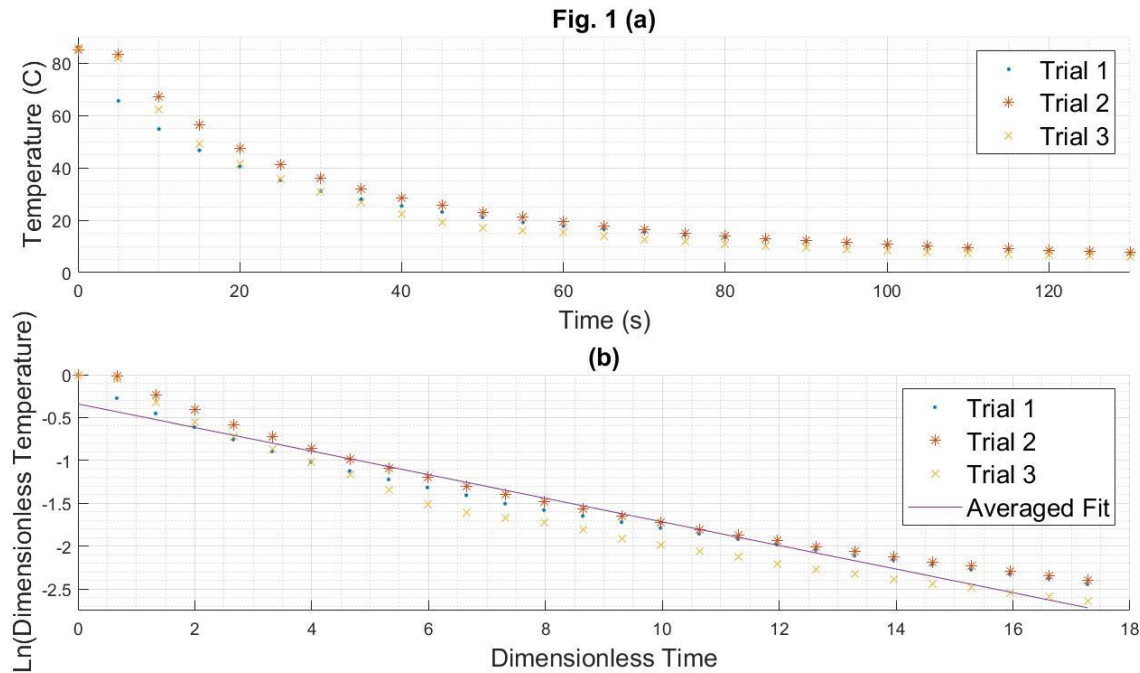


Figure 1: (a) The natural convection temperature response recorded by hand over three trials of an aluminum sphere of radius  $0.0254 \pm 0.0001$  m cooling in a bath of 0 °C stagnant ice-water. (b) The response plotted in dimensionless quantities and the average linear fit for all sets of data.

The same exact experiment was performed again but this time a pump was added into the ice-water bath to add some fluid velocity and cause forced convection to occur. The measured temperature response for three trails is plotted below in figure 2. By modeling the cooling with the lumped thermal capacity model, the biot number was found to be  $.197 \pm .002$  with equation 8. The forced convective heat transfer coefficient was then calculated to be  $1.58 \times 10^3 \pm 20 \frac{W}{m^2K}$  using equation 6.

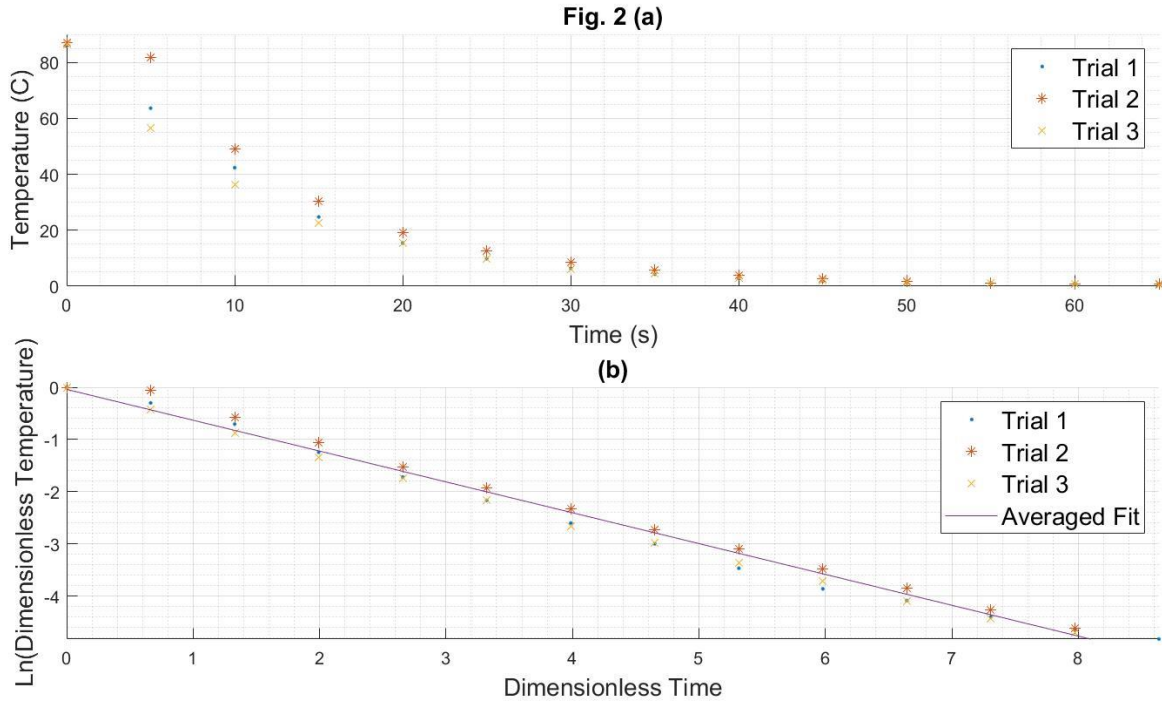


Figure 2: (a) The forced convection temperature response recorded by hand over three trials of an aluminum sphere of radius  $0.0254 \pm 0.0001$  m cooling in a bath of  $0^\circ\text{C}$  flowing ice-water. (b) The response plotted in dimensionless quantities and the average linear fit for all sets of data.

### Automatic Measurements

A slightly altered setup was used to automatically record the voltage data without having to physically read off the digital multimeter for every measurement. The aluminum sphere with the embedded thermocouple was connected as the input to a signal conditioning board. The output of the board was then routed through an operational amplifier to increase its magnitude and then inputted into an analog input channel on an Arduino microcontroller for data recording. The Arduino code attached in the appendix was used to output the measured voltage data to the serial port and then a terminal emulator program was used to log the data and save it for further data analysis. The measured temperature response for three trials is plotted below in figure 3. By modeling the cooling with the lumped thermal capacity model, the biot number was found to be  $.0366 \pm .0004$  using equation 8. The natural convective heat transfer coefficient was therefore calculated to be  $294 \pm 3 \frac{\text{W}}{\text{m}^2\text{K}}$  using equation 6.

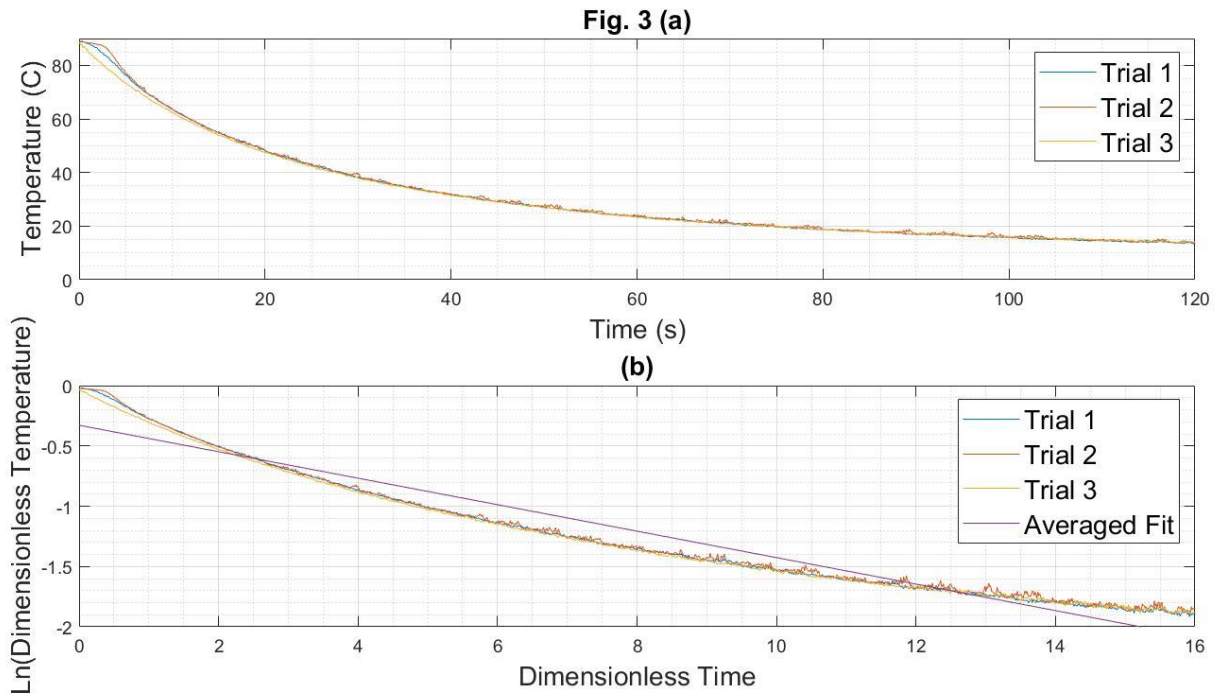


Figure 3: (a) The natural convection temperature response recorded by a microcontroller over three trials of an aluminum sphere of radius  $0.0254 \pm 0.0001$  m cooling in a bath of  $0^\circ\text{C}$  stagnant ice-water. (b) The response plotted in dimensionless quantities and the average linear fit for all sets of data.

The same exact experiment was performed again but this time a pump was added into the ice-water bath to add some fluid velocity and cause forced convection to occur. The measured temperature response for three trails is plotted below in figure 4. By modeling the cooling with the lumped thermal capacity model, the biot number was found to be  $.22 \pm .05$  with equation 8 and the convective heat transfer coefficient was then calculated to be  $1.7 \times 10^3 \pm 400 \frac{W}{m^2K}$  with equation 6.

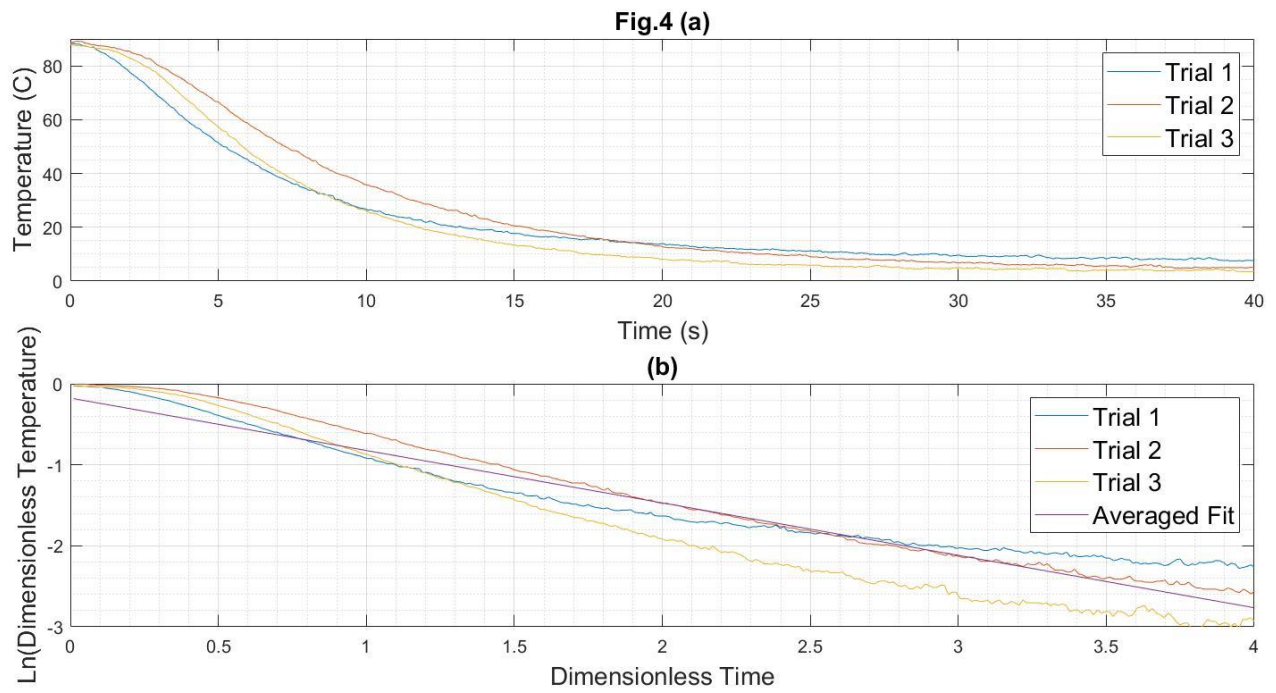


Figure 4: (a) The forced convection temperature response recorded by a microcontroller over three trials of an aluminum sphere of radius  $0.0254 \pm 0.0001$  m cooling in a bath of  $0^\circ\text{C}$  flowing ice-water. (b) The response plotted in dimensionless quantities and the average linear fit for all sets of data.

## Discussion:

The first law of thermodynamics is essential for developing a model of the temperature response of a fixed mass. It is easily stated as follows in equation 1 where E is the energy stored in the system, Q represents heat, and W represents work.

$$(1) \dots \Delta E = Q_{net\ in} - W_{net\ out}$$

It is important to note that this statement is merely a restatement of conservation of energy and all units are in joules. In a system where all the energy storage comes from a substance's internal energy (no kinetic or potential energy), the object being heated is an incompressible solid (like the metallic thermocouple junction sphere), and the dominant heat exchange method taking place is convection (an assumption that will need to be checked later), equation 1 can be simplified to equation 2.

$$(2) \dots hA(T_{solid} - T_{fluid}) = -\rho VC \frac{dT_{solid}}{dt}$$

Where h represents the convective heat transfer coefficient,  $T_{solid}$  and  $T_{fluid}$  are the temperatures of the solid and fluid respectively,  $\rho$  is the density of the object that's being examined, V is the object's volume, A is the object's surface area, and C is the object's specific heat that can be looked up in engineering tables. This is the governing differential equation for a solid object undergoing primarily convection and it can be solved to yield equation 3 for the temperature response.

$$(3) \dots \frac{T_{solid} - T_{fluid}}{T_0 - T_{fluid}} = \exp\left(-\frac{hA}{\rho VC} t\right)$$

This equation can more easily be used for a wide range of problems by converting surface temperature and time to non-dimensional surface temperature and time as shown in equations 4 and 5.

$$(4) \dots \theta_{solid} = \frac{T_{solid} - T_{fluid}}{T_0 - T_{fluid}}$$

$$(5) \dots \tau = \frac{k_{solid} t}{\rho C}$$

Where  $T_0$  is the initial temperature of the solid,  $k_{solid}$  is the thermal conductivity of the solid, and all other quantities remain the same as before. The equation for convective cooling in this form is what is known as the lumped thermal capacity model and a quantity known as the biot number can now be defined and used to validate the assumption that convection is the dominant form of heat transfer occurring.

Thermal resistances are quantities that define how much resistance heat will encounter from various modes of heat transfer when flowing from place to place. They can be used to quickly give a good idea of the modes of heat transfer that are dominating. Equations for these can be looked up for conduction and convection and are as follows:  $R_{cond} = \frac{L}{kA}$ , and  $R_{conv} = \frac{1}{hA}$  where L is the characteristic length of the problem and all other quantities remain the same<sup>1</sup>. Radiation can be ignored all together for temperatures this small because it will have a very small thermal resistance but to account for it one

would just look up the equation for radiative resistance and add it to that of conduction. The biot number is simply the ratio of the resistance from all other forms of heat transfer to that of convection and can now be calculated below in equation 6. The characteristic length for a sphere is its radius, R.

$$(6) \dots B_i = \frac{hR}{k}$$

The assumption that convection is dominating is valid if  $B_i \ll 1$  (the biot number is much less than one with a value smaller than .1 indicating less than a 5% error due to other modes of heat transfer being neglected), since the biot number is the ratio of all other heat transfer resistances to that of convection.

The overall equation can now be written in a very pleasing form in equation 7 known as the lumped thermal capacity model. This equation can be used to generate plots of  $\theta_{solid}$  vs  $\tau$  that can be used across a wide range of cooling problems in different systems of units due to the variables used being non-dimensional. Using this form of the equation can save an engineer a lot of time as a result. Values of temperature and time can just be converted to  $\theta_{solid}$  and  $\tau$ , and then dimensionless plots or tables can be used to quickly come up with solutions to problems.

$$(7) \dots \theta_{solid} = e^{-3B_i\tau}$$

By taking the natural log of both sides, the equation becomes that of equation 8.

$$(8) \dots \ln(\theta_{solid}) = -3B_i\tau$$

This relationship in equation 8 was plotted and used to find the biot number for every trial. In total four sets of three were used to obtain data for: manual data collection with free convection, manual data collection with forced convection, automatic data collection with free convection, and automatic data collection with forced convection. Table 1 shows all biot number data obtained.

Table 1: The obtained biot number data for all different trials of three with averages and standard deviations calculated. The lumped thermal capacity model was assumed to be valid when calculating all these values.

| Method of Data Collection and Experiment                       | Biot Number   |
|--|---------------|
| <b>Manual</b> data collection with <b>free</b> convection      | .046 ± .003   |
| <b>Manual</b> data collection with <b>forced</b> convection    | .197 ± .002   |
| <b>Automatic</b> data collection with <b>free</b> convection   | .0366 ± .0004 |
| <b>Automatic</b> data collection with <b>forced</b> convection | .22 ± .05     |

The most obvious thing from this data is that all values are well below 1, validating the lumped thermal capacity model's assumption that convection is the dominant mode of heat transfer. The values of free convection will have the least error (less than 5%) from neglecting all other modes of heat transfer since their biot numbers are less than .1. Therefore, the obtained values for free convection shall be trusted more than those of forced convection. This is since the biot number is directly proportional to h as can be seen in equation 6. Faster rates of cooling (such as those that one would expect from forced convection), result in a larger biot number and a less accurate model. Due to this fact, the lumped thermal capacity model is not valid for high-speed flows and another method to obtain convection data experimentally must be used in these cases.



The free convection trials have a percent error of 16.2% with each other whereas the forced convection trials differ by 11.7%. Some possible sources for this difference include the rate at which the metallic balls were lowered into the ice-water bath. This could cause a substantial difference in the biot numbers due to the nature of the metal cooling quicker near the start of the experiment when the temperature difference was the greatest. It is very hard to control for this variable since our setup required that the balls be taken out of the hot water bath and lowered into the cold water one all by hand. Also, depending on where the ball physically was at the moment defined as  $t=0$ , the curves can look substantially different near the beginning. All of figures 1-4 have some of their most prominent differences at very small  $t$  because of this effect. Trials with larger slopes around  $t=0$  were most likely more submerged in the ice-bath at the time designated  $t=0$ .

A mechanical system could be created to transfer the balls between baths, diminishing these effects. Another option to mitigate these effects is that the serial port or digital multimeter outputting the data and the physical setup could be recorded in the same frame so that the moment the ball is completely submerged could be more accurately linked to a data point. Analysis would then be started at this index and all prior indexes would be discarded. Recording experiments like this could be of use for experimentalists and engineers because referring to the video to help line-up data can eliminate some guess work and help achieve a greater level of accuracy.

The forced convection trials with automatic data collection had notably the biggest standard deviation. This was most likely since under forced convection, cooling happens rather quickly. If too much of a cooling curve is looked at in this instance, the lumped thermal capacity model breaks down and cooling is no longer purely logarithmic for large values of  $t$ . This becomes obvious in figures 3b and 4b when all dimensionless plots start bending upwards at large values of dimensionless time. Noise in the plots also gets amplified around these values. The automatic data collection system is unable to accurately measure small changes in temperature here due to underlying noise in the system. A possible way of mitigating this error would be to use a low-pass filter, that after carefully picking its cutoff frequency would diminish the sharp edges on the graphs at these large time values. Another possibility is to disregard these time intervals from data analysis entirely. The periods of more rapid cooling would then be the ones that determine the value of the linear fit and this would greatly affect the calculated values for the convective heat transfer coefficients. This dilemma of exactly which range of dimensionless time to choose when fitting the plots is also most likely the source of the majority of our error between the manual and automatic data transmission methods.

Another possible reason why the non-dimensional graphs become less linear at large values of non-dimensional time is that the value of  $h$  is dependent on the temperature difference to a degree and is therefore nonlinear over the range of temperatures tested. The problem could then be split up into multiple linear ranges and the biot numbers/convective heat transfer coefficients could be calculated for each range, ultimately yielding an equation for  $h$  as a function of temperature difference instead of a constant value.

The biot numbers can be used to calculate the values for the convective heat transfer coefficients using equation 6. The values for the thermal conductivity of aluminum and radius of the

Table 2: The obtained convection coefficient data for all different trials of three with averages and standard deviations calculated. In finding this data, the lumped thermal capacity model was assumed to be valid.

sphere can be found in the appendix. Table 2 tabulates these results with their according errors from the uncertainty of the radius measurement and the standard deviation of each data set's biot numbers.

| Method of Data Collection and Experiment                       | Convection Coefficient                   |
|--|--|
| <b>Manual</b> data collection with <b>free</b> convection      | $370 \pm 30 \frac{W}{m^2K}$              |
| <b>Manual</b> data collection with <b>forced</b> convection    | $1.58 \times 10^3 \pm 20 \frac{W}{m^2K}$ |
| <b>Automatic</b> data collection with <b>free</b> convection   | $294 \pm 3 \frac{W}{m^2K}$               |
| <b>Automatic</b> data collection with <b>forced</b> convection | $1.7 \times 10^3 \pm 400 \frac{W}{m^2K}$ |

As can be seen, all relative errors here are nearly identical to those found in the biot number calculations. This signifies that the error from the radius of the sphere is having very little effect on the final values of the convection coefficients. This makes since the sphere's radius was measured accurately to the nearest .1 mm. It is important to also note that all the previous reasons for errors in the biot numbers are still valid for each set's respective convection coefficient.

An online search reveals that normal free (natural) convection coefficients for waters and liquids are around 50 to 3,000  $\frac{W}{m^2K}$ . This value is between 50 to 10,000  $\frac{W}{m^2K}$  for forced convection in waters and liquids<sup>6</sup>. Our values fall into these ranges. It is hard to find specific values for these coefficients online because they depend on object geometry, Reynold's number (which is proportional to a fluid's velocity), and other quantities that are unique to every situation. Therefore, these are usually found through experimentation and checked against an acceptable range of values that can be found online<sup>5</sup>. All our obtained values fall into these acceptable ranges.

From the collected data, one can confidently say that cooling from forced convection in water under the conditions of this experiment happens much quicker than that of free convection—as a matter of fact it happened about five times quicker. A hypothesis can be made that generally, convection occurring when the fluid is moving quicker transfers more heat due to the movement of more molecules that can absorb and transfer energy across the cooling interface every second. This is therefore the reason why radiators of all kinds use fans to speed up air before it meets the radiator's surface. More experiments with different shapes and in different fluids would need to be conducted to definitively prove this hypothesis however.

## **Conclusion:**

This work examined the convective cooling of heated aluminum spheres under free and forced convection by measuring the temperature response with thermocouples and comparing that to theory described by the lumped thermal capacity model. Modeling convection accurately is essential for designing effective ways of cooling systems from PCs to power plants. Convection is a very complicated process to mathematically model through Newton's law of cooling because of the introduction of the convective heat transfer coefficient that is a function of many different variables and often not a constant for larger ranges of temperature differences. The lumped thermal capacity model can be used to obtain this value of  $h$  for problems in which convection is dominating the other modes of heat transfer. This can be of great value for developing mathematical models for cooling solutions to everyday devices exposed to fluids such as the atmosphere or a water source in the form of an ocean, river, or lake. It was found that the convection coefficient for water increases as fluid velocity increases. This was hypothesized to be generally true for convection but more experiments in different fluids and with differently shaped cooling surfaces are needed to completely verify this assumption. It was also discovered that the convective heat transfer coefficient is not necessarily a constant over a large range of temperatures. An improved model could account for these nonlinearities by assigning different constants to multiple different temperature ranges, doing enough experiments to be able to fit  $h$  to a function of the temperature difference, or by adding more higher order temperature difference terms (each with new constants) to Newton's law of cooling and solving for these through experimentation. Experimentation over larger ranges of temperatures could potentially make this problem with the model more obvious.

## Methods:

### Manuel Measurements

As depicted in figure 5, The k-type thermocouple used in this lab was referenced to 0 °C (cold junction) by sticking the reference junction in an ice-bath and allowing ample time for it to come to equilibrium. A digital multimeter was then hooked up across the spherical and reference thermocouple junctions to read the voltage difference across them. Any voltage offset on the digital multimeter when the spherical side was in the ice-water bath was removed and then the voltage reading when the spherical side was in a 90 °C water bath was recorded. These measurements were used to scale calibration data obtained online for a k-type thermocouple and then a linear fit was obtained as seen in figure 6.

While leaving the reference junction in the stagnant ice-water bath, the spherical end of the thermocouple was brought to 90 °C in a temperature-controlled water bath. It was then quickly transferred to and submerged in the cold bath. The digital multimeter was recorded with a smartphone and voltage readings were later written down by hand for five-second intervals after the hot sphere was submerged in this 0 °C bath for multiple minutes. This data was then taken into MATLAB, converted to temperature using the calibration relationship in figure 6, converted to non-dimensional quantities as discussed earlier in the lumped thermal capacity model, and plotted for all three free convection trials. From the set of three trails, biot numbers and convective coefficients were calculated along with their respective standard deviations.

A pump was carefully placed into the ice-water bath to cause forced convection via water movement and the same experiment and analysis was repeated. Careful

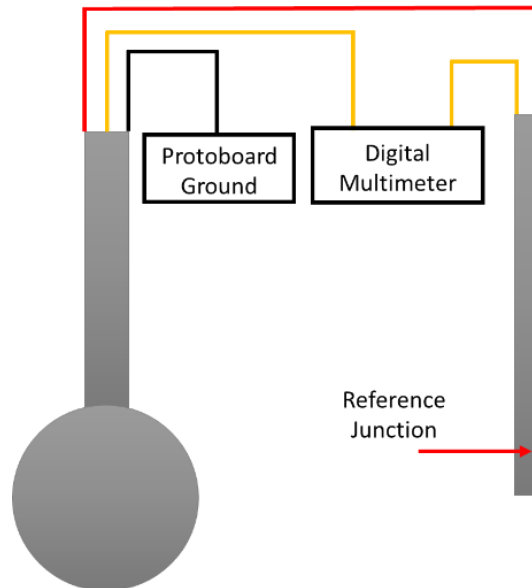


Figure 5: The setup used for manual data measurements that involves physically referencing one side of the thermocouple to an ice-water bath at 0 °C

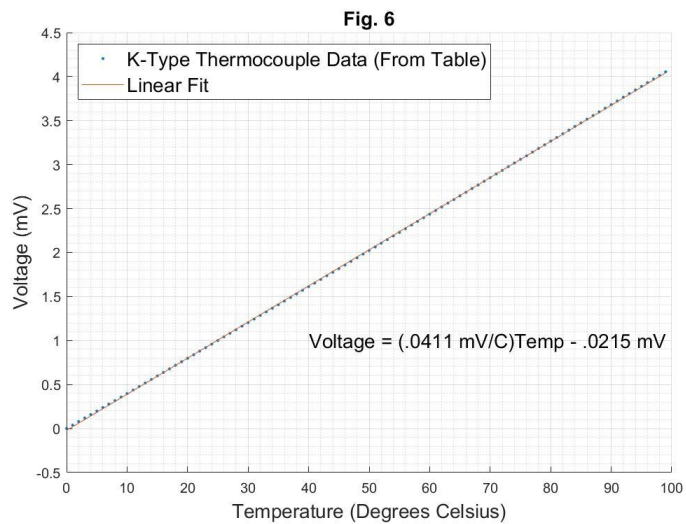


Figure 6: The calibration relationship used for a k-type thermocouple referenced to 0 °C over the range of temperatures in this experiment. The data in table 3 of the appendix was plotted and scaled accordingly based on calibration measurements at 0 °C and 90 °C.

consideration was given to keep the pump in the same exact position between trials since its direction and location can influence the convective heat transfer coefficient.

### Automatic Measurements

Figure 7 depicts the setup used for automatically recording data on a microcontroller (in this case an Arduino). The thermocouple output was passed into a signal conditioning board that was powered by +15 and -15V from a protoboard. The output from the signal conditioning board was then passed through an operational amplifier to ensure that voltage measurements took up as much of the Arduino's [0V 5V] range as possible before being inputted into the microcontroller for data collection. The Arduino was connected to a computer via USB and the code in the appendix was used to output data to the serial port that the device was connected to. A terminal emulator program named TeraTerm was used to log and save serial port data that the Arduino was printing out.

The same experiment as before was then conducted to find the biot number and convective coefficient for this setup. The aluminum sphere was brought to 90 °C in a temperature-controlled bath and then suddenly submerged in the 0 °C stagnant ice-water bath. This time however, data was logged using the new automatic setup. This data was then taken into MATLAB, converted to temperature using the calibration relationship in figure 6, converted to non-dimensional quantities using the lumped thermal capacity model, and plotted for all three free convection trials. From the set of three trails, biot numbers and convective coefficients were calculated along with their respective standard deviations.

A pump was then carefully placed into the ice-water bath to cause forced convection via water movement and the same experiment analysis was repeated. Careful consideration was given to keep the pump in the same exact position between trials since its direction and location can influence the convective heat transfer coefficient.

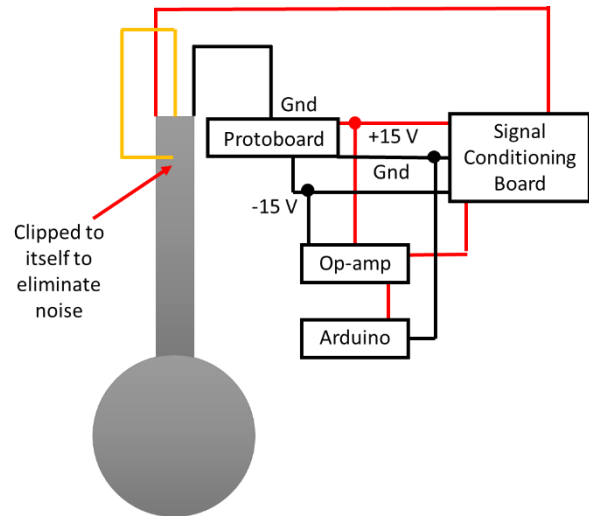


Figure 7: The setup used for automatic data measurements that involves using a referencing junction on a signal conditioning board. An operational amplifier with a gain of 4.7 was also used to obtain a greater accuracy in voltage measurements taken with the Arduino's limited resolution.

## Appendix:

### Arduino data acquisition code

```
float volts;
unsigned long int startTime=millis(), timeNow;
int samplePeriod=100; //sampling period in ms
void setup() {
Serial.begin(9600);
}
void loop() {
// put your main code here, to run repeatedly:
volts=analogRead(0)*5.0/1023.0;
timeNow= millis()-startTime;
Serial.print(timeNow);
Serial.print(", ");
Serial.println(volts,3);
delay(samplePeriod-1);
}
```

### Thermocouple calibration data

Table 3: Standard voltage table for a k-type thermocouple referenced to 0 °C over the range of temperatures used in this lab<sup>3</sup>. Example reading the table: A K-type thermocouple, referenced to 0 °C, would produce 2.727 millivolts at a temperature of 67 °C.

| °C | +1    | +2    | +3    | +4    | +5    | +6    | +7    | +8    | +9    |       |
|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0  | 0.000 | 0.039 | 0.079 | 0.119 | 0.158 | 0.198 | 0.238 | 0.277 | 0.317 | 0.357 |
| 10 | 0.397 | 0.437 | 0.477 | 0.517 | 0.557 | 0.597 | 0.637 | 0.677 | 0.718 | 0.758 |
| 20 | 0.798 | 0.838 | 0.879 | 0.919 | 0.960 | 1.000 | 1.041 | 1.081 | 1.122 | 1.163 |
| 30 | 1.203 | 1.244 | 1.285 | 1.326 | 1.366 | 1.407 | 1.448 | 1.489 | 1.530 | 1.571 |
| 40 | 1.612 | 1.653 | 1.694 | 1.735 | 1.776 | 1.817 | 1.858 | 1.899 | 1.941 | 1.982 |
| 50 | 2.023 | 2.064 | 2.106 | 2.147 | 2.188 | 2.230 | 2.271 | 2.312 | 2.354 | 2.395 |
| 60 | 2.436 | 2.478 | 2.519 | 2.561 | 2.602 | 2.644 | 2.685 | 2.727 | 2.768 | 2.810 |
| 70 | 2.851 | 2.893 | 2.934 | 2.976 | 3.017 | 3.059 | 3.100 | 3.142 | 3.184 | 3.225 |
| 80 | 3.267 | 3.308 | 3.350 | 3.391 | 3.433 | 3.474 | 3.516 | 3.557 | 3.599 | 3.640 |
| 90 | 3.682 | 3.723 | 3.765 | 3.806 | 3.848 | 3.889 | 3.931 | 3.972 | 4.013 | 4.055 |

Table 4: Properties of the aluminum metal used in the thermocouple-embedded spheres.

### Properties of Aluminum

| Material | $\rho$ (kg·m <sup>-3</sup> ) | $c$ (J/kgK) | $k$ (W/m K) |
|----------|------------------------------|-------------|-------------|
| Aluminum | 2707                         | 879         | 204         |

Aluminum Sphere radius = 0.0254 ± .0001 meters

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