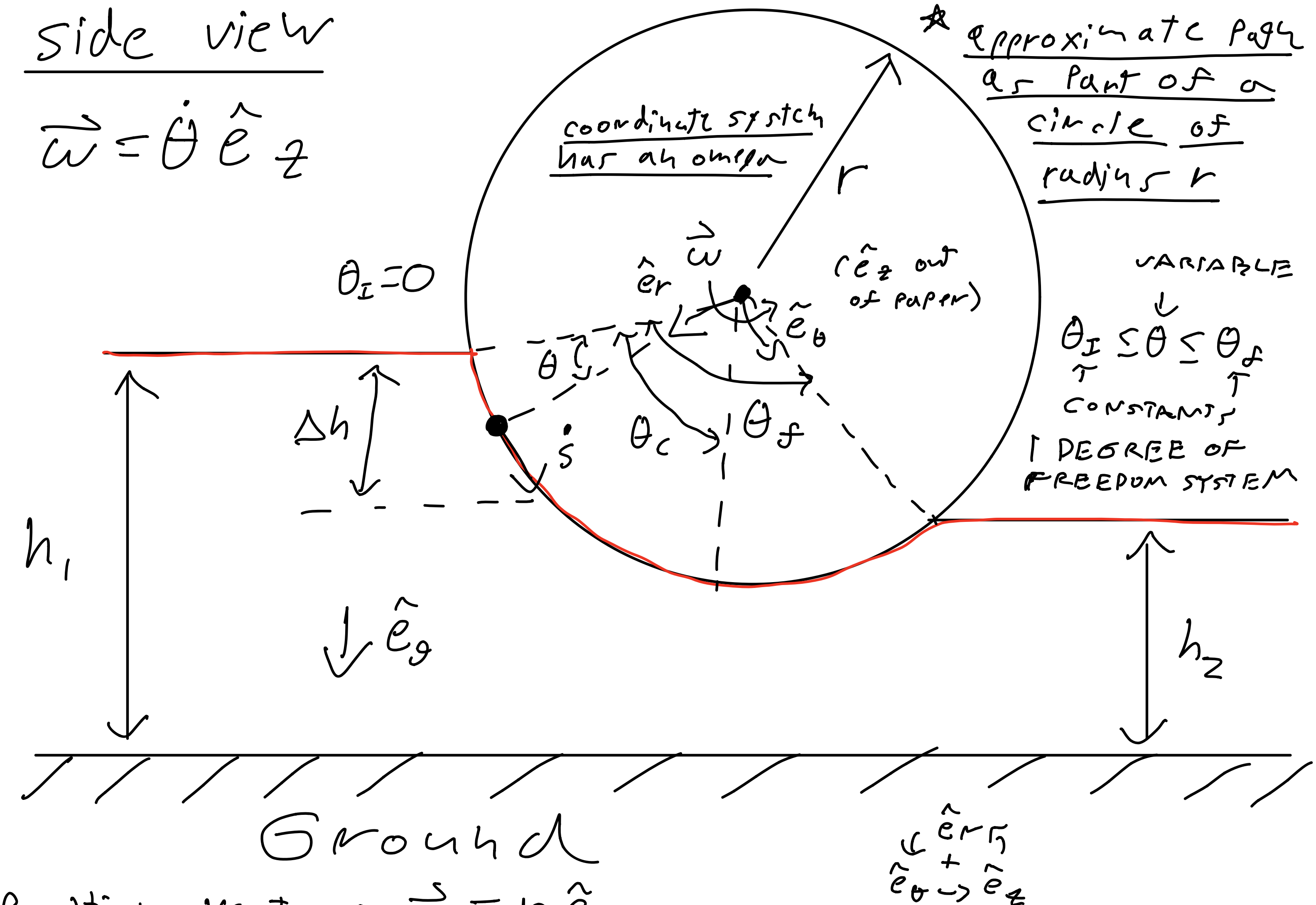
side view

$$\vec{\omega} = \dot{\theta}\,\hat{e}_z$$



Position vector: $\vec{r} = r\,\hat{e}_r$

$$\dot{\vec{r}} = r\dot{\hat{e}}_r = r(\omega \times \hat{e}_r) = r(\dot{\theta}\,\hat{e}_z \times \hat{e}_r)$$

$$= r\dot{\theta}\,\hat{e}_\theta$$

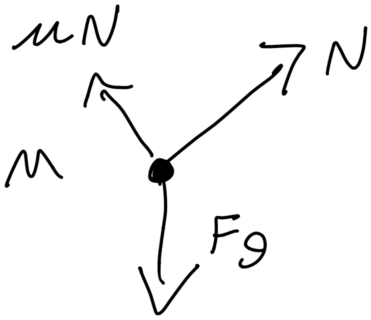
$$\ddot{\vec{r}} = r\ddot{\theta}\,\hat{e}_\theta + r\dot{\theta}\,\dot{\hat{e}}_\theta$$

$$= r\ddot{\theta}\,\hat{e}_\theta + r\dot{\theta}(\dot{\theta}\,\hat{e}_z \times \hat{e}_\theta)$$

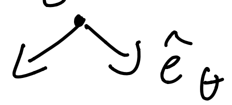
$$\ddot{\vec{r}} = r\ddot{\theta}\,\hat{e}_\theta - r\dot{\theta}^2\,\hat{e}_r$$

Now let's examine the forces on our cart that we are approximating as a point mass for simplicity.

⇓



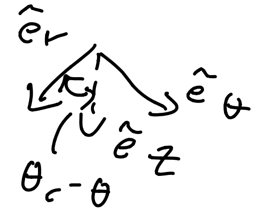
$\hat{e}_r$ $\Leftarrow$ $\hat{e}_z$ out of page



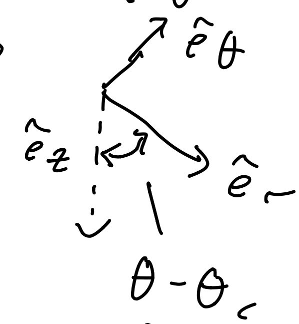
$\hat{e}_g$ always straight down.

when $\theta = \theta_c \Rightarrow \hat{e}_r = \hat{e}_g$

when $\theta < \theta_c \Rightarrow$



when $\theta > \theta_c \Rightarrow$



We need to project $\hat{e}_z$ onto $\hat{e}_\theta$ & $\hat{e}_r$ to exhibit this behavior.

$$\hat{e}_z = \underbrace{\cos(\theta_c - \theta)\hat{e}_r + \sin(\theta_c - \theta)}\hat{e}_\theta$$

careful examination of cases reveals that this expression for the projection works beautifully. ☺.

let's write sum of forces now

$$\vec{F} = -\mu N\hat{e}_\theta - N\hat{e}_r + mg\hat{e}_z$$

$$= -\mu N\hat{e}_\theta - N\hat{e}_r + mg\left[\cos(\theta_c - \theta)\hat{e}_r + \sin(\theta_c - \theta)\hat{e}_\theta\right]$$

$$\vec{F} = \left[mg\sin(\theta_c - \theta) - \mu N\right]\hat{e}_\theta + \left[mg\cos(\theta_c - \theta) - N\right]\hat{e}_r$$

Now we use; $\vec{F} = \frac{d\vec{p}}{dt} = m\ddot{\vec{r}}$ in this case $(F = ma)$

This gives us 2 equations of motion

in θ direction	in r direction
$mr\ddot{\theta} = mg\sin(\theta_c - \theta) - \mu N$	$-mr\dot{\theta}^2 = mg\cos(\theta_c - \theta) - N$

we now have a system of coupled differential equations that is nonlinear. oh goodie.

we will use numerical methods to solve this for given initial conditions.

θ-dir: $\ddot{\theta} = \frac{g}{r}\sin(\theta_c - \theta) - \frac{\mu N}{Mr}$

r-dir: $\dot{\theta}^2 = -\frac{g}{r}\cos(\theta_c - \theta) + \frac{N}{Mr}$

eliminate $\underline{N}$

$$\dot{\theta}^2 + \frac{g}{r}\cos(\theta_c - \theta) = \frac{-\ddot{\theta} + \frac{g}{r}\sin(\theta_c - \theta)}{\mu}$$

$$\mu\dot{\theta}^2 + \frac{\mu g}{r}\cos(\theta_c - \theta) = \frac{g}{r}\sin(\theta_c - \theta) - \ddot{\theta}$$

solve for $\theta, \dot{\theta}, \ddot{\theta}$ response

Now using θ-response solve for N.

$$N(t) = \left[\dot{\theta}^2 + \frac{g}{r}\cos(\theta_c - \theta)\right](Mr)$$

$\underline{22{,}000 \text{ N}}$

DIVIDE THIS BY HOW MANY WHEELS FOR WEIGHT EACH WHEEL IS carrying.