Alexander Ardalan

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Preparing Electrical Signals for Data Collection with Filters and Amplifiers

Abstract:

Engineers of all specialties will at times be faced with data sets that are non-ideal and in need of conditioning via electrical circuits, post-processing, or both to make data easier to extract meaning from. Low-pass or high-pass filters allow low or high frequency components respectively to pass through them relatively unaffected (passband) while drastically reducing the amplitude of other frequencies (stopband). These filters can be characterized by a time constant—a value that affects the response rate and passband to stopband transition point of the filter. Filters enable certain frequencies such as those of noise, offsets, or aliased frequencies to be targeted and reduced during data collection. Amplifiers increase the magnitude of signals with the energy from their own power supplies. This can allow more accuracy to be obtained when using lower resolution measuring devices or when dealing with small amplitude signals such of those coming from a thermocouple. These techniques can be used in combination along with averaging data in the frequency domain to obtain cleaner experimental or measured results.

Introduction:

Signal conditioning is an important aspect of experimentation that, if done incorrectly, can have a drastic negative effect on the quality of recorded data—making large sets of data unusable and amounting to many wasted man-hours. To correctly condition signals with electrical circuits, it is important to understand specific characteristics of common data collection circuit elements.

A simple RC filter circuit is comprised of a resistor, with resistance R, and capacitor, with capacitance C. The position of these two components relative to a filter's input and output signals determine if the filter will attenuate high frequency components of the signal (low-pass filter) or attenuate low frequency components of the signal (high-pass filter)[1]. The product of the chosen values of R and C define a quantity known as the RC time constant which has units of seconds and defines some important characteristics of the conditioning elements.

The value of RC is a measure of how quickly the filter will respond to a sudden change in voltage applied to the system—such as a square wave input where the signal varies almost instantaneously. One time constant after the initial application of a constant voltage to a RC filter that has been allowed to reach steady-state, the output signal of the filter will have recovered 63% of the change in the input's voltage. This value of RC characterizes how quickly the filtered signal will respond to changes in the states of the unfiltered signal[2].

A crucial quantity in signal conditioning is the cutoff, also known as the -3 decibel (dB) frequency or the corner frequency. This is the frequency at which a filter transitions from allowing a considerable amount of amplitude to pass through it (pass-band) to attenuating most of the amplitude it receives (stop-band) or vice versa. The gain of a filter is the ratio of the input to output magnitudes (in units of voltage) that a certain frequency experiences while passing through the component. The power gain is just this same ratio squared and has the property of being equal to .5 at the cutoff frequency of a filter. The magnitude of these gains often varies drastically over a large range of frequencies, so it is convenient to express them in a logarithmic scale, as the units of decibels does. Bode diagrams of the transfer function of a component are easy representations of how a filter will affect a range of frequency components passing through it. These plots are expressed as gain or power gain (in dB) vs frequency (in Hz or kHz)[1].

Inverting operational amplifiers are a circuit component whose frequency-response-bode-plots closely resemble that of a low-pass filter, however, low frequency signals have a gain greater than one but a flipped polarity. These devices will ultimately amplify the magnitude of a signal passing through them and need to be connected to an external power source to provide the energy input necessary to accomplish this task[3]. The cutoff frequency for an operational amplifier is defined as the frequency at which the power gain of the component is .5 that of its maximum DC value. As will be discussed later, this is the same as being 3 dB below the maximum decibel gain of the op-amp.

A combination of these conditioning elements is often used to target a certain range of frequencies that an engineer or system is interested in the most for data collection. The transfer function is overall a way of quantifying how different frequencies are affected by signal conditioning techniques. Different combinations of components will have unique transfer functions that will be a combination of the individual component's transfer functions. The overall transfer function is an important tool in designing a filtering circuit that fits the experiment. How does an engineer go about picking the specifics of these conditioning circuits that will best fit their intended purpose? This work focuses on revealing common techniques to obtain transfer function graphs for common filtering circuits and how to use them to improve the quality of collected data. In this article, the RC time constant was measured and used for a certain value of R and C in a filter. Frequency-response-bode-plots were generated for low-pass, high-pass, and operational amplifiers to see how they would affect different frequency input signals. Various methods were also used to determine a cutoff frequency for the different types of conditioning elements. How these phenomena can be combined and used together in engineering applications was also touched on when a high-pass filter and operational amplifier were used in series.

Results:

Characterizing an RC Filter

An RC low-pass filter comprised of a 9.96 kΩ resistor and a capacitor of unknown capacitance, C, was hooked up to a signal generator. The measured system response to a 100 Hz square wave input with a V_{pp} value of 5 V is depicted in Figure 1. The figure shows the input and output voltages of the filter with respect to ground as a function of time. It was measured that it took 720 μs for the output signal to recover 63% of the change in the input signal. As described by equation 1, this time value is what is known as the RC time constant and is the value of R and C multiplied together. Knowing the resistance and time constant, the capacitance was solved for and determined to be 72 nF. Setup diagrams for this and other parts of this article can be found in the methods section.

Sine waves of various frequencies and a V_{pp} value of 10 V were then sent through the same low-pass filter via the signal generator. Figure 2 shows the system's response to 3 different frequency sine waves. The -3 dB cutoff frequency was found by varying the input frequency coming from the signal generator until the oscilloscope measuring the system read an output voltage that was .707 times the input voltage (7.07 V) as described by equation 2. This locates the point at

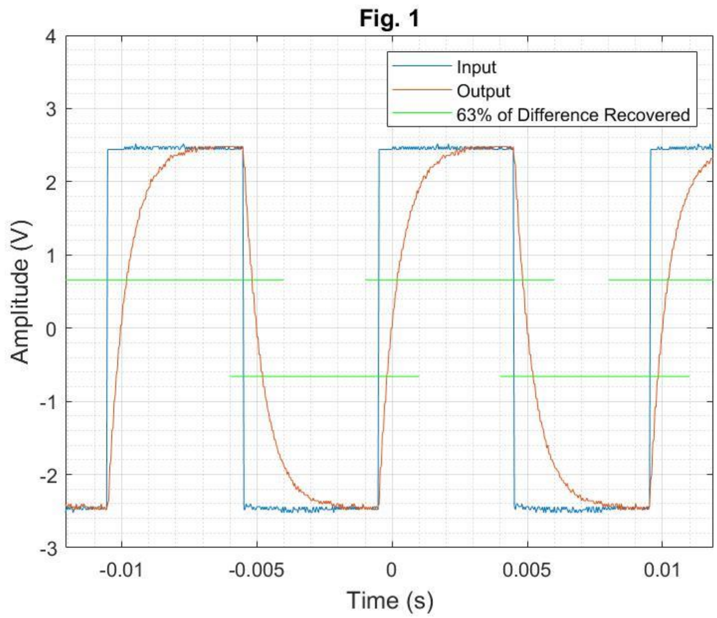

Figure 1: Response of a low-pass filter with resistance 9.96 kΩ and unknown capacitance to a 100 Hz square wave with a V_{pp} value of 5 V. The points at which 63% of the change in input voltage is recovered by the output signal of the filter are marked with horizonal lines.

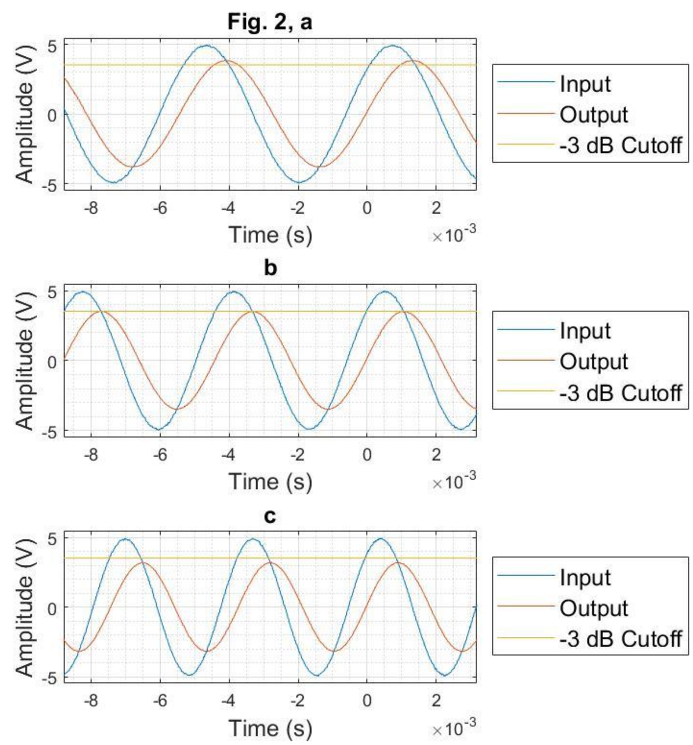

Figure 2: Response of same low-pass filter to varying input frequencies. (a) Response to 185 Hz. (b) Response to 228 Hz that was determined to be the cutoff frequency. (c) Response to 270 Hz.

which the output power is half that of the input and it occurred when the signal generator was making a signal of frequency 228 ± 3 Hz. Data was saved from multiple signals using this technique. The gains of these signals were calculated, converted to decibels with equation 2, and plotted as points on the filter's transfer function in figure 3 below.

As can be seen, it is hard to obtain enough data points using this method to get a good representation of the filter's response over a wide range of frequencies. New methods will now be looked at to help improve upon this dilemma.

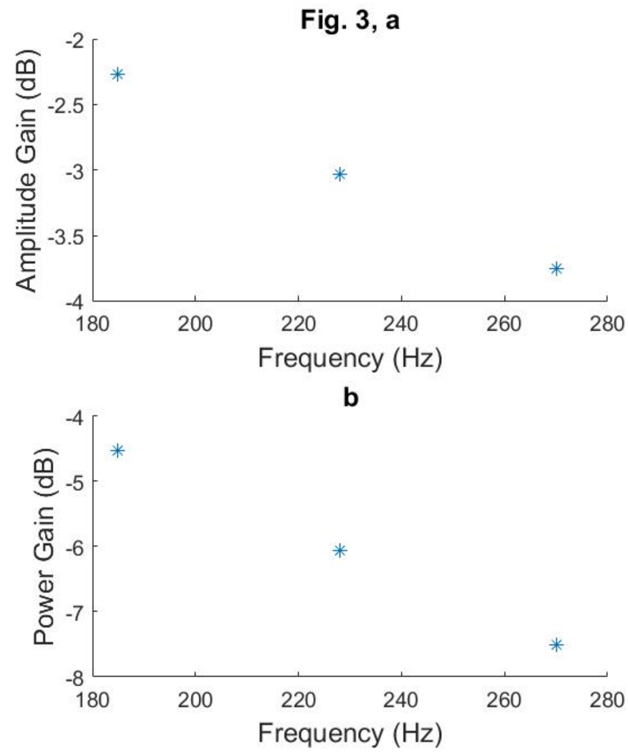

Figure 3: Gains of Figure 2 responses calculated and put on frequency response plots. (a) Voltage gain as a function of frequency. (b) Power gain as a function of frequency.

The positions of the capacitor and resistor in the following circuit were then switched, forming a high-pass filter. The signal generator was set to give the filter an input of white noise (a random signal having equal intensity at different frequencies) and the filter's input and output voltage vs time responses were each recorded eight different times. This data was taken into MATLAB and the fast Fourier transform (code can be found in the appendix) was done on all the data sets to take them into the frequency domain. Gains were then calculated by dividing output by input amplitudes in the frequency domain and the eight gains were averaged in this domain. Figure 4 shows the resulting frequency response of this filter over a large range of different input frequencies. A discontinuity of sorts was observed around 60 Hz and the -3 dB cutoff was measured to happen at 230 ± 5 Hz.

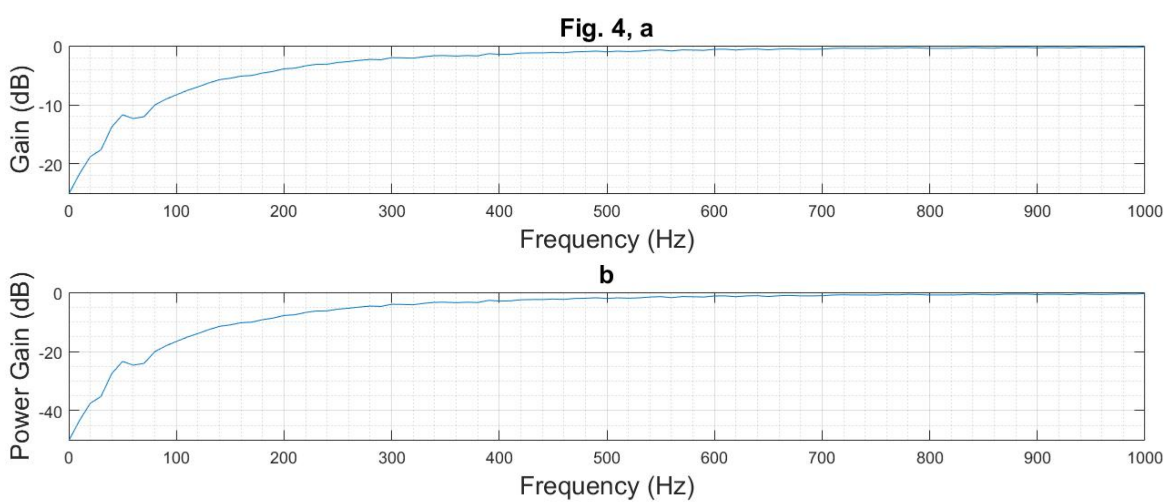

Figure 4: High-pass filter's frequency response to white noise. A discontinuity of sorts was observed around 60 Hz and the -3 dB cutoff was measured to happen at 230 ± 5 Hz. (a) Graph of voltage gain vs frequency. (b) Graph of power gain vs frequency.

Characterizing an Inverting Operational Amplifier

Next, an inverting amplifier was setup with a 1 kΩ and 49.9 kΩ resistor as described in the methods section. A computer program was then used to generate continuous white noise with a digital acquisition board that was sent through the op-amp. The digital acquisition board and computer program also was used to measure the voltage vs time responses of the op-amp's input and output. The computer program took the discrete Fourier transform of both signals 10 different times and averaged them in the frequency domain. The gain was then calculated, converted to decibels, and plotted in figure 5 that shows the resulting frequency response of this conditioning component. The -3 dB cutoff frequency was found to be 14.2 ± .2 kHz. The operational amplifier's DC gain was found to be 35 ± 3 dB.

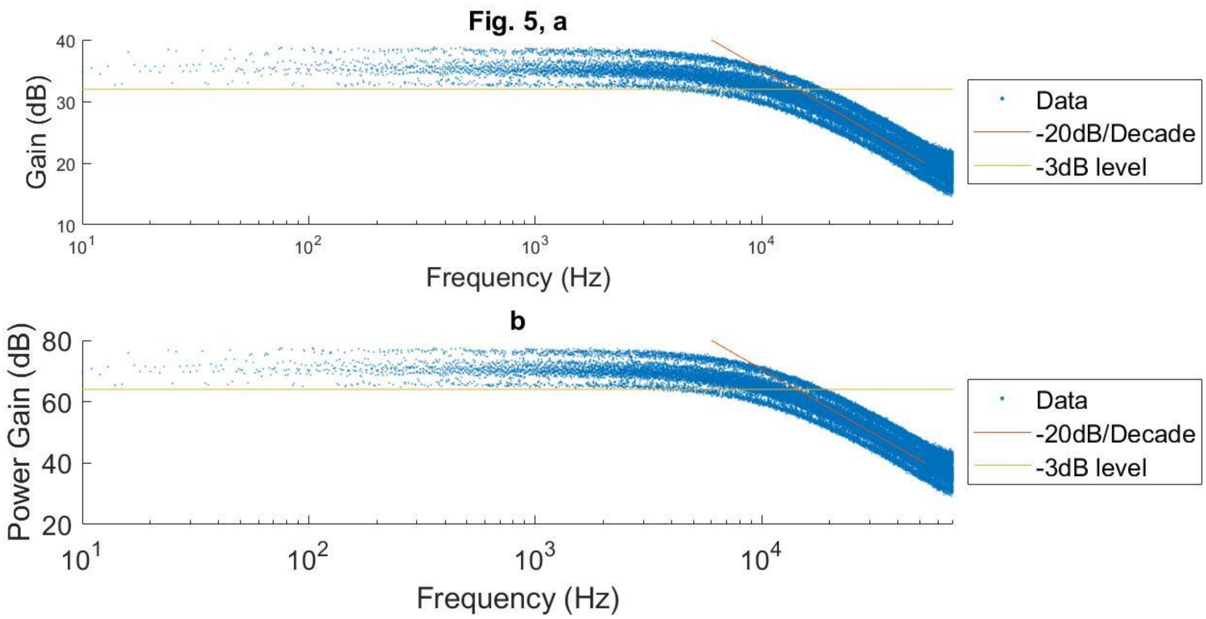

Figure 5: The described operational amplifier's frequency response over a large range of frequencies with a fit applied to the -20dB/decade falloff and -3 dB level graphed. (a) Graph of voltage gain vs frequency. (b) Graph of power gain vs frequency.

Characterizing an Inverting Operational Amplifier and High-Pass Filter in Series

Now that the individual frequency responses of the high-pass filter and inverting op-amp have been examined, what do you think happens if a high-pass filter and inverting op-amp are placed in series? This was done with the operational amplifier depicted in Figure 5 and the high-pass filter depicted in figure 4. The resulting transfer function was obtained using the same exact method as was for the operational amplifier and is graphed in Figure 6 below. Two -3 dB cutoff frequencies were found at 14.3 ± .2 kHz and 2.3 ± .2 kHz. The maximum DC gain of this combination was found to be 34 ± 3 dB.

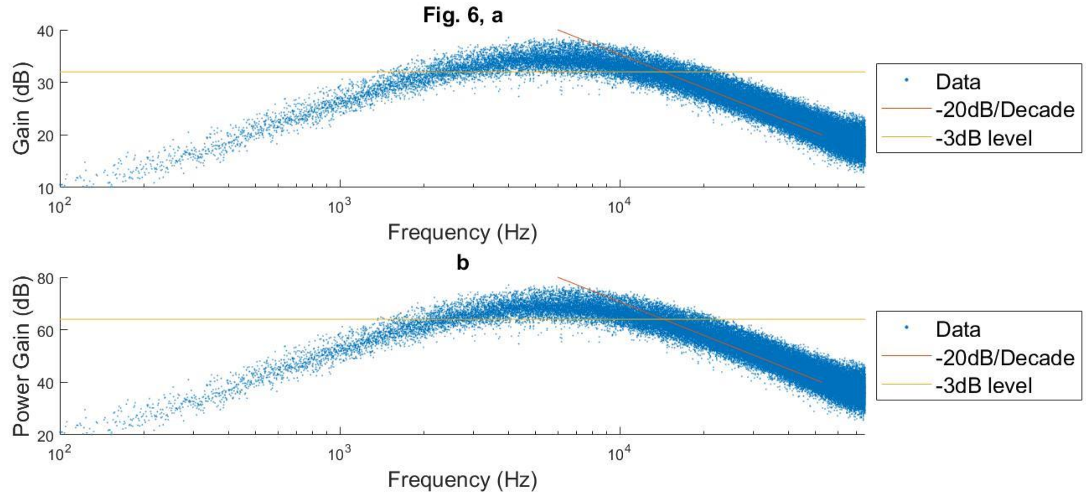

Figure 6: The frequency response of the inverting op-amp and high-pass filter in series plotted over a large range of frequencies with a fit applied to the -20dB/decade falloff and the -3 dB level graphed. (a) Graph of voltage gain vs frequency. (b) Graph of power gain vs frequency.

Discussion:

RC filters have a couple defining features that can be used to characterize them or match them to a need. It was observed that the value of R times C had a physical meaning. This has to do with the rate at which the output of the filter catches up to the input. The voltage on the non-grounded side of a capacitor in an RC filter follows equation 1 bellow.

$$(1) \ldots V_C = V_0(1 - e^{\frac{-t}{RC}})$$

Where V_0 is the filter's input voltage, R is the resistance of the filter's resistor, and C is the capacitance of the filter's capacitor. Plugging in a value of $t = RC\ seconds$ yields $V_C = .63V_0$. Since $V_C = V_{out}$ and $V_0 = V_{in}$, this means that after a time of one time constant, 63% of the difference between input and output has been recovered. It is important to note that this is only true if the voltages have reached equilibrium and then the input voltage is changed suddenly, such as what happens in a square wave of a frequency much less than $\frac{1}{RC}\ Hz$ (a signal with a period much greater than the value of multiple time constants).

Equation 1 can be interpreted as the capacitor introducing lag into the system. The system no longer reacts instantaneously to a varied voltage input. Instead it reacts on the order of the RC time constant. This should be considered when selecting resistance and capacitance values of an RC filter for signal conditioning. As was done in the lab, by measuring this time constant by looking for the $V_{out} = .63V_{in}$ relationship on a voltage vs time plot, it is possible to solve for the value of R or C if the other is known. Measuring the time constant is depicted in figure 1.

Looking at the response of filters in the frequency domain—by taking the discrete Fourier transform of recorded time domain signals—yields completely different looking graphs. The ratio of $\frac{V_{out}}{V_{in}}$ is known as the filter's gain and this is a function of frequency alone. The -3dB cutoff frequency of a RC filter (low-pass or high-pass) is defined as the point at which the signal's power is halved from input to output. Since the amplitude (or voltage) squared is proportional to the power of the signal, this means that this happens when the gain is $\frac{1}{\sqrt{2}}$. If we define a decibel (dB) as a unit with the relationship expressed in equation 2, it is possible to solve for the decibel value at which the gain of the system is equal to $\frac{1}{\sqrt{2}}$.

$$(2) \ldots dB = 20\log_{10}(gain)$$

Plugging $gain = \frac{1}{\sqrt{2}}$ into the above equation yields a value of -3.01 dB. The frequency at which this happens is of importance because it defines the frequency at which the filter switches from dramatically attenuating a signal, to letting most of it pass through or vice versa. As can be seen in our results, the positions of the resistor and capacitor in a RC filter determine if it will behave as a device that lets frequency components higher (high-pass) or lower (low-pass) than the cutoff frequency pass through it relatively unaffected. The cutoff frequency is also defined by the mathematical relationship given in equation 3.

$$(3) \ldots f_{cutoff} = \frac{1}{2\pi RC}$$

This equation can be used to calculate the theoretical cutoff frequency for our measured RC time constant of 720 µs. Plugging into equation 3, this theoretical frequency is 221 Hz. The measured cutoff frequency was determined to be 228 ± 3 Hz, yielding a minimum percent error of 1.81%. This is well within an acceptable margin of error and the results agree with theory quite well. Comparing theory to the 230 ± 5 Hz measured cutoff obtained from the transfer function of the high-pass filter made from the same capacitor and resistor also gives a minimum error of 1.81%. It makes sense how this value also agrees with theory since the cutoff frequency of a filter is independent of whether it is in a high or low pass configuration. These cutoff frequencies can be used to eliminate wide ranges of unwanted frequencies via filters before collecting data from a system or experiment.

Operational amplifiers are very complicated devices that consist of many different transistors and resistors that produce a desired overall effect. Due to the fact these devices produce gains whose magnitudes are greater than 1—in other words they amplify a given input signal—they need to be connected to their own DC power source to put this additional energy into the system (since power is proportional to voltage squared). The absolute difference between these two power supply voltages given to the operational amplifier is directly related to the maximum voltage it can output. If the op-amp is setup in such a way that calls for gain amplification that would result in a voltage beyond this point, it will become saturated at its maximum possible output voltage, unable to achieve a voltage higher than the one determined by its power source.

The gain of an operational amplifier can be selected by using equation 4 in combination with figure 9 in the methods section to solve for relative values of R_f and R_{in}. Resistors can then be picked so that they satisfy this relationship to give the desired gain.

$$(4) \dots \; Desired\ Gain = -\frac{R_f}{R_{in}}$$

The operational amplifier used in this lab had an R_f value of 49.9 kΩ and an R_{in} vale of 1 kΩ, therefore its theoretical gain value was 49.9. In decibels, this is a gain of 34 dB. The average DC gain value of this component was measured in figure 5 to be exactly this. Although it is important to note that this gain consistently fluctuates by up to about 3 dB in each direction from its average value. This could be due to the amplifier's gain being slightly dependent upon the input voltage since the white noise given to the op-amp comprised of multiple different amplitude signals for every frequency.

An operational amplifier also behaves somewhat like a low pass filter as can be seen in figure 5. It amplifies lower-frequency components at a constant gain, until is hits a cutoff frequency, at which the gain amplification drops off at a rate of approximately -20db/decade. This unit of decade represents a factor of 10 increase and can be easily represented as each large division if the linear scale of the frequency axis is changed to a logarithmic scale as it was in figures 5 and 6. It is important to be aware of this drop-off effect if during data collection the frequency of a component that is being ran through the op-amp lies beyond the cutoff frequency. A way that this can be corrected is by utilizing the gain bandwidth product of an op-amp. The gain bandwidth product is defined by equation 5.

$$(5) \dots \; Gain\ Bandwidth\ Product = (Gain)(f_{cutoff})$$

This product has the feature of always being a constant value for a certain operational-amplifier. This gives us an inverse relationship between the gain and cutoff of an op-amp. Due to this fact, the cutoff frequency can be increased to ensure that the proper range of frequencies (bandwidth) is amplified. Decreasing the amplifier's gain by selecting different resistors will in turn increase the cutoff frequency and the -20 db/decade falloff of the amplifier will happen at a higher frequency.

The cutoff frequency of our operational amplifier was measured to be 14.2 ± .2 kHz. Applying equation 5 with this value and the chosen gain of 49.9, the constant gain bandwidth product of our amplifier is 709 kHz. Comparing this value to the one of 800 kHz for 25 degrees Celsius given in the Texas Instruments LM 148 op-amp data sheet yields a percent of nearly 13%[4]. This error is most likely since the operational amplifier was running hotter than the temperature of the room. This would make sense because as temperature increases, the gain bandwidth product drops off linearly for this op-amp. This can be seen in the provided graphs in the op-amp's data sheet. To accurately compare this value the temperature of the operational amplifier should have been recorded.

Putting the high-pass filter in series with an operational amplifier has the effect of multiplying their frequency response graphs together as can be seen in figure 6. This is a useful technique if it is desired that only a certain band of frequencies (bandwidth) pass through the filter relatively unattenuated. Using this technique, it is possible to remove a DC-low-frequency offset, amplify a signal to levels above the resolution of a piece of equipment, and remove high-frequency noise from a signal with only two components: a high-pass RC filter and an operational amplifier. The measured bandwidth defined by the two -3 dB cutoff frequencies of this configuration was [2.3 ± .2 kHz to 14.2 ± .2 kHz]. It is important to note that the cutoff frequency of the op-amp stayed the same, while the cutoff frequency of the high-pass filter is shifted up to a higher frequency than the one that equation 3 gives us. This has to do with the fact that the transfer functions are being multiplied together, and the new -3 dB point that the filter is contributed is measured as -3 dB from the maximum DC gain of the operational amplifier. This needs to be remembered when creating bandwidth filters that amplify and restrict recorded data to a band of targeted frequencies.

Conclusion:

RC filters and operational amplifiers can be used alone or together to increase the quality of measured data during an experiment. Often experimentalists are limited by the resolutions, ranges, and sensitivities of their measuring devices. Conditioning signals so that they can be measured adequately is just as important as knowing the limitations of your measuring devices. High-frequency noise or aliased-frequencies contaminating data can be eliminated by implementing a low-pass filter. Offsets that could potentially put signals outside of a device's measuring range can be removed from data by using high-pass filters. Operational amplifiers can amplify signals by a set amount to increase their visibility to obtain more accuracy if a measuring device has a low resolution or sensitivity. Lastly, these methods can be used in combination to achieve any desired effect. The topics discussed in this article are heavily used by cutting-edge experimentalists and any system that needs to obtain constant high-quality data from its surroundings such as self-driving cars or self-landing rockets.

Methods:

Characterizing an RC Filter

A simple low-pass filter as shown in figure 7 was created with a resistor rated as 10 kΩ and a random capacitor. The resistance of this component was then checked with a multimeter to obtain a more accurate value for its resistance. A 100 Hz square wave generated by a signal generator was then sent through the filter. A 2-channel oscilloscope was hooked up as shown in figure 8 to measure the voltage at the filter's output and input nodes relative to ground. The oscilloscope was then used to measure the time it took for the output voltage to recover 63% of the change in an input voltage. This number was recorded, and the signal was saved and brought into MATLAB for further analysis. The recorded time value represents the time constant—value of resistance times capacitance—of the filter. Knowing the resistance, the capacitance can now be solved for because their product is equal to the time constant.

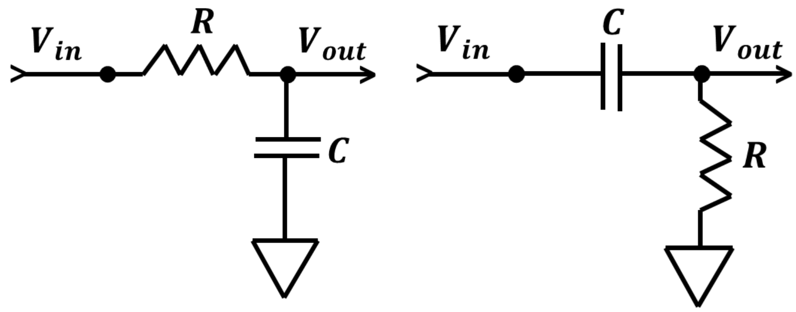

Figure 7: The configuration for (Left) low-pass and (right) high-pass RC filters examined in this article.

Now the cutoff frequency was found by passing sine waves of varying frequencies through the conditioning element via the signal generator until the oscilloscope measured a gain of $\frac{1}{\sqrt{2}}$. This cutoff frequency value was recorded. The gains of a couple other frequencies were also measured, converted to decibels with equation 2, and plotted as points on the transfer function of the component in MATLAB.

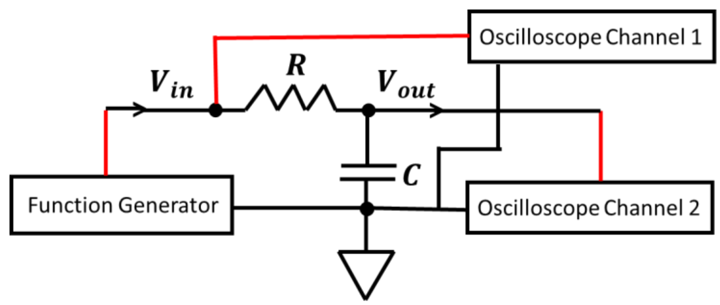

Figure 8: The configuration used to pass signals of varying frequencies through the low-pass filter and measure the input's and output's voltage responses.

Next a method for obtaining a complete transfer function quickly was tried. The setup was used as in figure 8, but the resistor and capacitor were switched to form a high-pass filter. The signal generator was used to pass white noise through the component and the oscilloscope was used to record the response of the input and output nodes eight different times. These saved signals were taken into MATLAB where the discrete Fourier transform was performed on them to take them into the frequency domain. The code for the discrete Fourier transformation is attached in the appendix. After this transformation, the signals were averaged in the frequency domain and then the resulting frequency dependent gains were, calculated, converted to decibels with equation 2, and plotted. The power gain is just the normal gain ratio squared and was also calculated and plotted in decibels.

Characterizing an Inverting Operational Amplifier

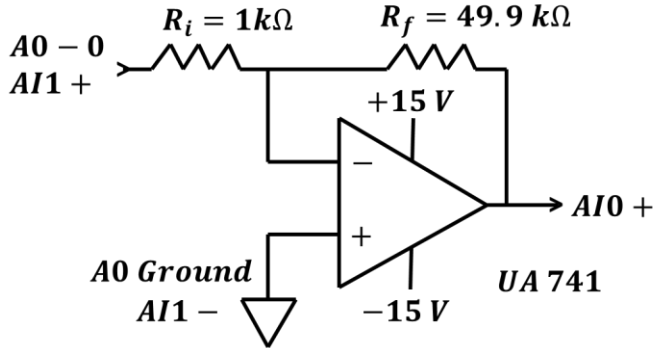

Figure 9: The configuration used to pass white noise through the operational amplifier and measure the input and output voltage responses with respect to ground. A digital acquisition board was used where AIX stands for analog input channel X and AO analog output channel X.

A digital acquisition board was hooked up to an operational amplifier with the setup shown in figure 9 to provide a signal and to measure the system's response. Two different channels of the digital acquisition board were used to measure the input and output voltages of the op-amp with respect to ground. A computer program was made to generate a white noise input signal and record the amplified and unamplified responses in the time domain. The program also took the discrete Fourier transforms of both signals for a set number of samples and plotted their averaged frequency domain amplitudes. These averaged frequency domain graphs and the time domain plots of the last signal to be averaged were saved and loaded into MATLAB for further analysis. In MATLAB, the gain was calculated by dividing the amplified amplitudes by their unamplified counterparts. The gain and power gain were then converted to decibels and plotted on a dB vs frequency plot with a logarithmic frequency scale. A fit was then applied to the -20dB/decade 'falloff' section of the graph and a line was plotted that represented the -3dB cutoff point. The intersection of these two lines was determined to be the cutoff frequency.

Characterizing an Inverting Operational Amplifier and High-Pass Filter in Series

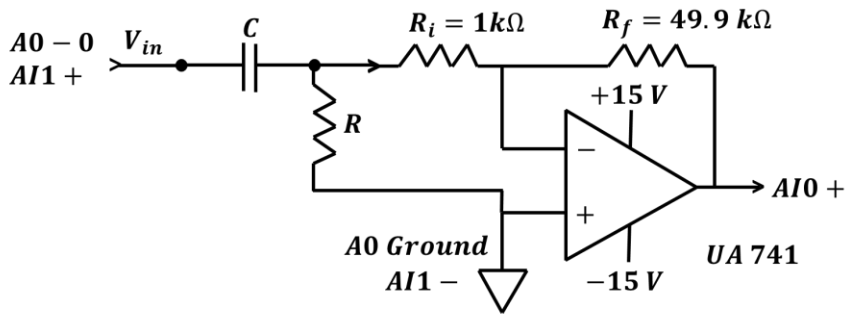

Figure 10: The configuration used to pass white noise through the operational amplifier and high-pass filter and series. The configuration's input and output voltages with respect to ground were also measured in a similar manner as before with a digital acquisition board.

The same exact setup was used as with the operational amplifier alone, but now a high-pass filter was placed in series with the op-amp as shown in figure 10. The setup was given an input of white noise and the input and output responses were measured all with the same data acquisition board. The same computer program and MATLAB techniques were used to acquire, analyze, and represent the system's response. With the transfer function plotted, it became apparent that the system had two -3dB frequencies. Both frequencies, defining the bandwidth—band of frequencies that dominates the output of the system—were recorded.

Appendix:

Discrete (Fast) Fourier Transform MATLAB Script

```
clc;
clear all ;+5
close all ;
%This script generates a 20 Hz size wave of unit amplitude, of duration 1
%second, and time resolution of 1E-2 seconds; then calculates the
%single-sided Fourier transform and plots both quantities
dt=1E-2; %s
T=1; %s
f_sig=20; %Hz
7
t=[0:dt:T-dt]; %time vector
y=sin(2*pi*f_sig*t); %amplitude vector
subplot(121)
plot(t,y, '-b' , 'LineWidth' ,2); %plot the time series
xlabel( 'time (s)' );
ylabel( 'Amplitude (V)' );
title( 'Time domain signal' )
set(gca, 'FontSize' ,20, 'LineWidth' ,2)
F = fft(y); %calculate the Fast Fourier Transform
Fs=1/dt; %define the sampling frequency
L=length(t); %define the length of your time and amplitude vectors
P2 = abs(F/L); %take the normalized amplitude of your Fourier transform
P1 = P2(1:L/2+1); %adjust the length and scale to get single sided spectrum
P1(2:end-1) = 2*P1(2:end-1);
f = Fs*(0:(L/2))/L; %create your frequency vector
subplot(122)
plot(f,P1, '-r' , 'LineWidth' ,2); %plot the Fourier transform
xlabel( 'frequency (Hz)' );
ylabel( 'Amplitude (V)' );
set(gca, 'FontSize' ,20, 'LineWidth' ,2)
title( 'Fourier Transform' )
xlim([0 50])
```

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